

## OPTIMAL CONTROL VIA THE INITIAL CONDITION IN A HYPERBOLIC PROBLEM

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ABSTRACT. In this article, we consider a problem of optimal control of the initial condition in a hyperbolic problem. We get the existence, and uniqueness of the optimal solution to this problem. The gradient of the cost functional on the set of admissible controls is derived via the solution of the adjoint problem. We establish the necessary optimality condition.

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