

## PSEUDOSOLITONS ON THE GENERALIZED CONFORMABLE KORTEWEG-DE VRIES EQUATION

J. NOYOLA-RODRÍGUEZ, J.C. HERNANDEZ GÓMEZ, S. GATICA RAMOS  
AND JAIR J. PINEDA-PINEDA

**ABSTRACT.** We consider the Korteweg-de Vries (KdV) model with classical derivative for outflows on water surfaces where the medium is assumed to be shallow and replace the classical first order derivative in time by the generalized conformable fractional derivative of order  $\alpha$  with  $0 < \alpha \leq 1$  and different kernels  $T(t, \alpha)$  defining fractional derivative mentioned above. We prove the existence of new soliton-like traveling waves which we call *pseudodolitons* propagating in a not necessarily linear wavefront. We study the dynamical system associated to the equation with generalized conformable fractional derivative and calculate some balance laws for the energy. In general it is concluded that the pseudosolitons have two main characteristics depending on the kernel of the fractional derivative. If the fractional derivative is conformable then the pseudosolitons propagate over large distances preserving their shape and amplitude and converge to the soliton-like solutions of the classical KdV when  $\alpha \rightarrow 1$ . On the other hand, if the derivative is nonconformable then the pseudosolitons only preserve the properties of the soliton-like traveling waves.

### REFERENCES

- [1] T. Abdeljawad: *On conformable fractional calculus*, J. Comput. Appl. Math., **279**(2015), 57-66.
- [2] Mark J. Ablowitz and Harvey Segur: *Solitons and the Inverse Scattering Transform*, Society for Industrial and Applied Mathematics, 1981 (SANM).
- [3] R. Abreu-Blaya, J.E. Nápoles, R. Reyes, J.M. Rodríguez and J.M. Sigarreta: *On the conformable fractional logistic models*, Math. Methods Appl. Sci., **43**(2020), No. 7, 4156-4167.
- [4] O.P. Agrawal: *A new Lagrangian and a new Lagrange equation of motion for fractionally damped system*, J. Appl. Mechanics, **68**(2001), No. 2, 339-341.
- [5] A. Atangana, D. Baleanu and A. Alsaedi: *New properties of conformable derivative*, Open Math., **13**(2015), 889-898.
- [6] A. Fleitas., J.E. Nápoles., José. M. Rodríguez., J.M. Sigarreta Almira: *Note on the generalized conformable derivative*, Rev. Un. Mat. Argentina, **62**(2021), 443-457.
- [7] M. García and G. Omel'yanov: *Interaction of solitary waves for the generalized KdV equation*, Commun. Nonlinear Sci. Numer. Simul., **17**(2012), No. 8, 3204-3218.
- [8] C. S. Gardner, J. M. Green, M. D. Kruskal and R. M. Miura: *Method for solving the Korteweg-de Vries equation*, Phys. Rev. Lett., **19**(1967), 1095.
- [9] R. Gorenflo and F. Mainardi: *Fractional Calculus: Integral and differential equations of fractional order*, in *Fractals and Fractional Calculus in Continuum Mechanics* (A. Carpinteri and F. Mainardi (Eds)), Springer, 1997 (CISM 378), pp. 223-276.

---

*Received:* November 19, 2023. *Revised:* February 15, 2024.

*2020 Mathematics Subject Classification:* 35A35.

*Key words and phrases:* KdV equation, soliton, fractional derivative, pseudosoliton, dynamic system, conformable derivative.

- [10] R. Khalil, M. Al Horani, A. Yousef and M. Sababheh: *A new definition of fractional derivative*, J. Comput. Appl. Math., **264**(2014), 65-70.
- [11] A.A. Kilbas, H.M. Srivastava and J.J. Trujillo: *Theory and Applications of Fractional Differential Equations*, Elsevier, 2006.
- [12] S. Momani: *An explicit and numerical solutions of the fractional KdV equation*, Math. Comput. Simulation, **70**(2005), 110-118.
- [13] S. Momani, Z. Odibat and A. Alawneh: *Variational Iteration Method for Solving the Space-and Time-Fractional KdV Equation*, in *Numerical Methods for Partial Differential Equations*, Wiley InterScience, 2008, pp. 262-271.
- [14] I. Mustafa, M. Parto-Haghighi, M. Ali Akinlar and C. Yu-Ming: *New numerical solutions of fractional order Korteweg-de Vries equation*, Results in Physics, **19**(2020), Article ID 103326, 5 pages.
- [15] J. Noyola-Rodríguez and G. Omel'yanov: *Numerical and asymptotic description of solitons for GkdV equations with nonhomogeneous nonlinearities*, J. Math. Sci. Adv. Appl., **48**(2017), 27-46.
- [16] K.B. Oldham and J. Spanier: *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, Elsevier, 1974.
- [17] G. Omel'yanov: *Collision of solitons for a non-homogenous version of the KdV equation: asymptotics and numerical simulation*, in *Numerical Analysis and its Applications* (I. Dimov, I. Faragó and L. Vulkov (Eds)), Springer, 2017, pp. 517-524.
- [18] G.A. Omel'yanov and M.A. Valdez-Grijalva: *Asymptotics for a  $C^1$ -versión of the KdV equation*, Non-linear Phenomena Complex Syst., **17**(2014), 106-115.
- [19] P. Veerasha, D.G. Prakasha and S. Jagdev: *Solution for fractional forced KdV equation using fractional natural decomposition method*, AIMS Mathematics, **5:2**(2019), 798-810.
- [20] N.J. Zabuski: *A synergetic approach to problems of nonlinear dispersive wave propagation and interaction*, in *Nonlinear Partial Differential Equations* (W. F. AMES (Ed.)), Academic Press, 1967, pp. 223-258.
- [21] N.J. Zabuski and M.D. Kruskal: *Interactions of "solitons" in a collisionless plasma and the recurrence of initial states*, Phys. Rev. Lett., **15**(1965), No. 6, 240-243.

Universidad Autónoma de Guerrero  
 Facultad de Matemáticas. Ext. Acapulco  
 Carlos E. Adame 54, 39650 Acapulco de Juárez, Guerrero, México  
 E-mail address: 20264@uagro.mx

Universidad Autónoma de Guerrero  
 Facultad de Matemáticas. Ext. Acapulco  
 Carlos E. Adame 54, 39650 Acapulco de Juárez, Guerrero, México  
 E-mail address: jcarloshg@gmail.com

Universidad Autónoma de Guerrero  
 Facultad de Matemáticas. Ext. Acapulco  
 Carlos E. Adame 54, 39650 Acapulco de Juárez, Guerrero, México  
 E-mail address: 13342327@uagro.mx

Benemérita Universidad Autónoma de Puebla  
 Instituto de Ciencias (IC)  
 Ecology and Survival of Microorganisms Research Group  
 Laboratorio de Ecología Molecular Microbiana  
 Puebla 72570, México

§  
 Universidad Autónoma de Guerrero  
 Facultad de Matemáticas. Ext. Iguala  
 Carretera Federal Iguala-Taxco, Iguala de la Independencia 40000, México  
 E-mail address: jpineda@uagro.mx