

NEW CLASSES OF GENERAL QUASI VARIATIONAL INCLUSIONS

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Dedicated to our respected, honorable, visionary and motivated Professors

ABSTRACT. In this article, we introduce and consider some new classes of general quasi variational inclusions, which provide us with unified, natural and simple framework to consider a wide class of unrelated problems arising in pure and applied sciences. We prove that the general quasi variational inclusions are equivalent to the fixed point problems. This alternative formulation is used to discuss the existence of a solution as well as to propose some iterative methods. Convergence analysis is investigated under certain mild conditions. Since the general quasi variational inclusions include quasi variational inequalities, variational inequalities, and related optimization problems as special cases, our results continue to hold for these problems. It is an interesting problem to compare these methods with other technique for solving quasi variational inclusions for further research activities.

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