

MATHEMATICAL ANALYSIS OF MALARIA TRANSMISSION  
DYNAMICS WITH IMPULSIVE RELEASE  
OF *WOLBACHIA*-INFECTED MALE MOSQUITOES

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ABSTRACT. In this paper, we formulate a mathematical model of malaria transmission dynamics with impulsive and periodic release of *Wolbachia*-infected male mosquitoes. Our objective is to evaluate the impact of periodic release of *Wolbachia*-infected male mosquitoes on malaria transmission. Subsequently, to find out how often and in what quantities *Wolbachia*-infected male mosquitoes need to be released to effectively reduce malaria transmission. By using the comparison principle, Floquet theory and some of the analytical methods, we obtain the basic reproduction number. Then, we demonstrated that the disease-free periodic solution is globally stable if the reproduction number is less than unity, and we prove the uniform persistence of the disease if the reproduction number is greater than unity. Finally, we perform numerical simulations to illustrate our theoretically pertinent results.

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