J. Adv. Math. Stud.
Vol. **17**(2024), No. 2, 226-236
http://journal.fairpartners.ro

SOME SPECIAL TRANSFORMATIONS FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS USING EXACT SOLUTION METHODS

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Abstract. Nonlinear partial differential equations (NPDEs) are used to model many phenomena in applied sciences such as physics, wave propagation, heat transfer, mass transport, fluid mechanics, solid mechanics, materials science, geophysics, chemistry, biology, and economics. There are many methods based on the Ansatz method to find exact solutions for NPDEs. Most of these methods share some particular common steps to derive algebraic equation systems, except for the auxiliary functions used. Some of these common steps are the reduction of nonlinear partial differential equations (NPDEs) to ordinary differential equations (NODEs), and the determination of the balance parameter for the proposed solution form. With the development of new and powerful computers in recent years, obtaining solution sets of algebraic equation systems required cumbersome and tedious algebraic calculations can be easily performed using computer algebra systems (CAS). On the other hand, these aforementioned common steps to obtain algebraic equation systems still require computation by hand. However, these steps can be difficult to implement due to the variety of terms in NPDEs. To our knowledge, there is no study that presents techniques to overcome this difficulty. Therefore, the main contribution of this study is to provide a literature review for researchers to analyze NPDEs that contain different types of terms and propose auxiliary transformations that are used to find exact solutions.

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Received: September 27, 2023. *Revised*: February 12, 2024.
2020 *Mathematics Subject Classification*: 35B08, 35C08, 76B15.
Key words and phrases: Ansatz method, auxiliary transformation, exact solution method, $\tan h$-function method, $\left(\frac{G'}{G}\right)$-expansion method.

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