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GENERALIZED RATIONAL TYPE CONTRACTIONS AND DISCONTINUITY AT FIXED POINT

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Abstract. In this paper we give some common fixed point theorems under a more generalized contractive condition which admit discontinuity at the fixed point. Our results include many well known theorems which provide the answers to the question of continuity of contractive mappings at their fixed points. Some examples are also given to illustrate our results. Moreover, an application of obtained results to threshold activation functions is accomplished.

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