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MULTIVALUED GENERALIZED α-NONEXPANSIVE MAPPINGS IN HYPERBOLIC SPACES: SPK-ITERATIVE APPROACH

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Abstract. This paper focuses on the SPK iterative algorithm for approximating the fixed points of multivalued generalized α-nonexpansive mappings within uniformly convex hyperbolic spaces. Additionally, we demonstrate both strong and Δ-convergence results for this class of mappings. A numerical example is provided to illustrate our findings. Additionally, we demonstrate the application of these results to fractional differential equations and equivalent nonlinear integral equations.

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