

HIGHER ORDER STRONGLY UNIFORM CONVEX FUNCTIONS

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ABSTRACT. Some new concepts of the higher order strongly uniform convex functions with an increasing modulus $\phi(\cdot)$ vanishing only at 0 are considered in this paper. Some properties of the higher order strongly uniformly convex functions are investigated under suitable conditions. The parallelogram laws for Banach spaces are obtained as applications of higher order strongly affine uniform convex functions as novel applications. It is shown that the minimum of the higher order strongly uniform convex functions can be characterized by the variational inequalities. Some important special cases as applications of our results are discussed. Results obtained in this paper can be viewed as refinement and improvement of previously known results.

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