

**ON HYERS-ULAM STABILITY OF TWO SINGULAR FRACTIONAL  
INTEGRO-DIFFERENTIAL EQUATIONS**

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ABSTRACT. The concept of stability for differential equations guarantees convergent of the numerical sequences in the process of obtaining solutions for the problems. Researchers usually have to consider heavy conditions to conclude stability of some differential equations and this be use in the software programs. As you know, it appears considerable errors in some computer calculations for obtaining numerical solutions of singular differential equations and one can not get its exact reason because it is a very technically note in the software. Thus, it is important we focus on stability of singular fractional integro-differential equations. In this work, we investigate Hyers-Ulam stability of two singular fractional integro-differential equations with usual and integral boundary conditions under some conditions.

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