

**ON THE SOME COHOMOLOGICAL PROPERTIES
OF BANACH ALGEBRAS**

HOSSEIN EGHBALI SARAI AND KAZEM HAGHNEJAD AZAR

ABSTRACT. In this paper, we establish some relationships between the Connes-amenability and super-amenability of Banach algebras and we obtain some new results in the cohomological properties of group algebras. We have some results in the weak amenability and strongly Connes-amenability of Banach algebras.

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University Mohaghegh Ardabili
Faculty of Science
Department Mathematics and Applications
Str. Daneshgah, No. 1, Code: 0451, Ardabil, Iran
E-mail address: h.eghbali.sarai@gmail.com

University Mohaghegh Ardabili
Faculty of Science
Department Mathematics and Applications
Str. Daneshgah, No. 1, Code: 0451, Ardabil, Iran
E-mail address: haghnejad@uma.ac.ir