

HANKEL DETERMINANT FOR CLASS OF ANALYTIC FUNCTIONS INVOLVING q -DERIVATIVE OPERATOR

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ABSTRACT. Quantum theory is an important tool to deal with complicated and difficult information. The notion of quantum theory has wide application in many fields like special functions and quantum physics and so on. Thus motivated by this, we proposed the upper bound for third Hankel Determinant (HD) of analytic functions by using q -differential operator and related with Cassinian ovals $(x^2 + y^2) - 2(x^2 - y^2) = q^2 - 1$, which reduces to Lemniscate of Bernoulli for $q \rightarrow 1^-$. The proposed approach is also compared with some existing methods to show the reliability and effectiveness of the proposed methods. At last, we investigated several interesting properties of these newly defined class, which is useful for future directions.

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