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ON APPROXIMATION RESULTS BY MODIFIED BERNSTEIN OPERATORS VIA (p,q)-CALCULUS

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Abstract. In the present paper, some approximation properties of the modified Bernstein operators based on (p,q)-calculus are examined. Firstly, we computed the moments up to fourth order and also the second and fourth central moments are obtained. Next, we proved Korovkin's type approximation theorem. Further, we examined the order of approximation by means of the modulus of continuity and the rate of convergence for a function f on Lipschitz class. Moreover, we present the Voronovskaya-type asymptotic theorem for the related operators.

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