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NEW CHARACTERIZATIONS OF HIGHER ORDER STRONGLY log-CONVEX FUNCTIONS

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Abstract. Some new concepts of the higher order strongly log convex functions involving an arbitrary bifuction are considered in this paper. Some properties of the higher order strongly convex functions are investigated under suitable conditions. The optimality conditions for the differentiable higher order strongly log-convex functions are characterized by variational inequalities. It is proved that the parallelogram laws for Banach spaces can be obtained as novel applications of higher order strongly affine convex functions. Some important special cases are discussed. Results obtained in this paper can be viewed as refinement and improvement of previously known results.

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