

**EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR NONLINEAR
MIXED VOLTERRA-FREDHOLM INTEGRO-DIFFERENTIAL
EQUATIONS**

AHMED A. HAMOUD, AMOL D. KHANDAGALE AND KIRTIWANT P. GHADLE

ABSTRACT. This paper establishes some important latest innovations in the existence and uniqueness results by means of Krasnoselskii's fixed point and the Banach fixed point theorems for Caputo fractional Volterra-Fredholm integro-differential equations with initial condition. New conditions on the nonlinear terms are given to pledge the equivalence. Finally, an illustrative example is also presented.

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Taiz University
Department of Mathematics
Taiz, Yemen
E-mail address: drahmedselwi985@gmail.com

Dr. Babasaheb Ambedkar Marathwada University
Department of Mathematics
Aurangabad, India
E-mail address: kamaldsk@gmail.com

Dr. Babasaheb Ambedkar Marathwada University
Department of Mathematics
Aurangabad, India
E-mail address: drkp.ghedle@gmail.com