SOLVABILITY OF FUNCTIONAL EQUATIONS VIA FIXED POINT THEOREM OF L-FUZZY SET-VALUED MAPS

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ABSTRACT. In this paper, a fixed point theorem for L-fuzzy set-valued map defined on a complete metric space is established by taking a general contractive inequality. It is shown herein that a few known significant metric fixed point theorems in the framework of fuzzy set-valued and multivalued mappings can be easily followed from our main result. A nontrivial example is provided to validate the hypotheses of our established idea. From application point of view, we study solvability conditions of a class of functional equations arising in dynamic programming, by using the techniques of L-fuzzy set-valued maps.

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