

2-STRUCTURED OPTIMIZATIONS

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ABSTRACT. In this article, we have developed a generalized optimization model for 2-structure by using some fundamental techniques of functional analysis. Some necessary and sufficient conditions (NASC) for a ϵ -controllable problem are deduced in arbitrary 2-Banach spaces. The existence and uniqueness of solutions are discussed without assuming any type of normality conditions. Also, a complete characterization of the solution is given in terms of a hyper-plane.

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