

GENERALIZED VECTOR EQUILIBRIUM PROBLEMS AND NONSMOOTH VECTOR OPTIMIZATION

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ABSTRACT. Some properties of pseudoconvex functions w.r.t. an equilibrium function, defined by means of limiting subdifferential, are obtained. Furthermore, the equivalence between generalized Minty vector equilibrium problems (in short, GMVEPs) (resp., generalized Stampacchia vector equilibrium problems (in short, GMVEPs)) involving limiting subdifferential and nonsmooth vector optimization problems (in short, VOPs) are studied under pseudoconvexity condition.

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