

GENERAL LYAPUNOV TYPE INEQUALITY FOR SUGENO INTEGRAL

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ABSTRACT. In this paper, Lyapunov type inequality for the Sugeno integral on an abstract spaces is studied in a rather general form, thus closing the series of papers on the topic dealing with special cases restricted to the (pseudo-)additive operation. Also, several examples are given to illustrate the validity of this inequality. Moreover, a strengthened version of Lyapunov type inequality for Sugeno integrals on a real interval based on a binary operation \star is presented.

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Received: February 11, 2017. Revised: September 18, 2017.

2010 Mathematics Subject Classification: 03E72, 26E50, 28E10.

Key words and phrases: Sugeno integrals, Lyapunov inequality, fuzzy integral inequality, non-additive.

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