

ON A HYBRID SHRINKING PROJECTION METHOD IN BANACH SPACES

YING LIU AND HANG KONG

ABSTRACT. In this paper, we introduce a hybrid shrinking projection method for finding a common element of the set of common fixed points for a finite family of closed quasi- ϕ -nonexpansive mappings, the set of solutions of a generalized mixed equilibrium problem and the set of solutions of a variational inequality problem for a pseudomonotone Lipschitz mapping in a uniformly smooth, 2-uniformly convex Banach spaces. Then, under appropriate conditions, we prove the sequence generated by the hybrid shrinking projection method converges strongly to a particular element of the common solution set of the above three problems. Our results improve and unify some existing results.

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Hebei University
College of Mathematics and Information Science
Baoding, 071002, China
E-mail address: ly_cyh2013@163.com

Hebei University
College of Mathematics and Information Science
Baoding, 071002, China
E-mail address: 1969849957@qq.com