

## NONEXISTENCE RESULTS FOR A SEMI-LINEAR EQUATION WITH FRACTIONAL DERIVATIVES ON THE HEISENBERG GROUP

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**ABSTRACT.** The first major topic of this paper is to establish a Fujita-type theorem for the blow-up of solutions to a semi-linear wave equation with fractional damping in the Heisenberg group. Our proof is based on the test function method. Then the situation is extended to systems of equations of the same type. This results extend previous works.

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