J. Adv. Math. Stud.
Vol. **16**(2023), No. 1, 01-14
http://journal.fairpartners.ro

RECONSTRUCTION OF TOPOLOGICAL SPACES FROM n POINTS DELETED SUBSPACES

A. JOSEPHINE SHILPA DEVI AND S. MONIKANDAN

Abstract. The n-deck of a topological space X is the set

$$\mathcal{D}_n(X) = \{[X - \{x_1, x_2, \ldots, x_n\}] : x_1, x_2, \ldots, x_n \in X\},$$

where $[Z]$ denotes the homeomorphism class of Z. A space X is called topologically n-reconstructible if whenever $\mathcal{D}_n(X) = \mathcal{D}_n(Y)$, then X is homeomorphic to Y. The *multi n-deck* of a space X is the set that contains all n-cards of X (that is, it includes all homeomorphism types of n-cards). The space X is n-reconstructible from its multi n-deck, we will say it is *weakly n-reconstructible.* It is shown that the topological properties T_i are n-reconstructible for $i = 0, 1, 2, 3$, and 2-reconstructible for $i = 3\frac{1}{2}, 5, 6$. It is also proved that the weight of a space and the number of isolated vertices in a T_1 space are 2-reconstructible. Also we prove all Hausdorff spaces with a compact subspace obtained by deleting n points, all compact Hausdorff spaces whose reconstructions are compact, all T_1 spaces with a finite number of isolated points, the discrete space, the space of rationals and the space endowed with the countable complement topology are n-reconstructible. Finally, we show the property that a bounded ordered space attaining its bounds to be path connected is weakly 1-reconstructible, and that a space to be hyper-connected and an Alexandroff space to be ultra-connected are weakly 2-reconstructible.

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Received: June 22, 2022. *Revised*: November 13, 2022.
2010 *Mathematics Subject Classification*: 54B99, 05C60, 54B05, 54A05, 54Dxx.
Key words and phrases: Reconstruction, k-reconstruction, homeomorphism.

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Manonmaniam Sundaranar University
Mathematics Department
Tirunelveli – 627 012, India
E-mail address: ashilpadevi1995@gmail.com

Manonmaniam Sundaranar University
Mathematics Department
Tirunelveli – 627 012, India
E-mail address: monikandans@gmail.com