

FIXED POINT THEOREMS FOR MIZOGUCHI-TAKAHASHI
RELATION-THEORETIC CONTRACTIONS

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ABSTRACT. The purpose of this paper is to prove a relation-theoretic version of the famous fixed point result of Mizoguchi and Takahashi in the metric spaces endowed with an arbitrary binary relation. The fixed point result of this paper generalizes the famous results of Nadler [7], Mizoguchi and Takahashi [6] and several other fixed point results for set-valued mappings defined on the spaces endowed with various kinds of binary relations.

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