

EXPONENTIALLY GENERAL VARIATIONAL INEQUALITIES

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ABSTRACT. Variational inequalities provide us with a unified, natural, novel and simple framework to study a wide class of problems arising in pure and applied sciences. In this paper, we introduce and consider a new class of exponentially general variational inequalities. Some new iterative methods are proposed and analyzed using the auxiliary principle technique. Our results present a significant improvement of previously known results for solving variational inequalities and related optimization problems. Several open problems have been suggested for further research in these areas.

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