

**SOME RESULTS ON  $\psi$ -HILFER VOLTERRA-FREDHOLM FRACTIONAL  
INTEGRO-DIFFERENTIAL EQUATIONS**

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ABSTRACT. This paper is devoted to investigate the existence, uniqueness, estimates on the solution and continuous dependence of solution of Volterra-Fredholm fractional integro-differential equations involving  $\psi$ -Hilfer fractional derivative. Our results are based on the Banach fixed point theorem and integral inequality of mixed type.

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