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BOUNDS FOR THE MEAN INTEGRAL OF THE POWER RECIPROCAL OF A FUNCTION UNDER DIFFERENT CONVEXITY CONDITIONS

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Abstract. This article contributes to the theory of convexity by addressing a previously unexplored issue: how to determine bounds for the mean integral of the power reciprocal of a function under two different notions of convexity: the classical and trigonometric notions. Some of the derived bounds are obtained using general techniques supported by auxiliary results, while others are more directly related to the specific structure of the convexity inequalities. These bounds are explicit and analytically tractable. Detailed proofs are provided throughout, with all secondary results clearly referenced.

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