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ON GLOBAL EXTENSION PROBLEM FOR HOM-LIE SUPERALGEBRAS

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Abstract. Let $(\mathcal{L}, [\,,\,], \alpha)$ be a Hom-Lie superalgebra, $\mathcal{E} := \mathcal{E}_{\bar{0}} \oplus \mathcal{E}_{\bar{1}}$ a $\mathbb{Z}_2$-graded vector space and $\pi \colon \mathcal{E} \longrightarrow \mathcal{L}$ an even epimorphism of $\mathbb{Z}_2$-graded vector spaces. This paper deals with the global extension problem for Hom-Lie superalgebras which asks to describe and classify all Hom-Lie superalgebra structures that can be defined over $\mathcal{E}$ so that π becomes an epimorphism of Hom-Lie superalgebras. Hom crossed product is introduced and used to give a classifying object for the global extension problem: given a Hom-Lie superalgebra $(\mathcal{L}, [\,,\,], \alpha)$ and a $\mathbb{Z}_2$-graded vector space $\mathcal{V} := \mathcal{V}_{\bar{0}} \oplus \mathcal{V}_{\bar{1}}$, a cohomological object $\mathbb{GH}^2(\mathcal{L}, (\mathcal{V}, \gamma))$ is constructed and we show that it classifies all Hom-Lie superalgebras that have an epimorphism of Hom-Lie superalgebras on $\mathcal{L}$ with kernel of dimension $\dim(\mathcal{V})$. Co-flag Hom-Lie superalgebra is introduced and classified.

REFERENCES

[1] A. Agore and G. Militaru: *On a type of commutative algebras*, Linear Algebra Appl., **485**(2015), 222-249.
[2] A. Agore and G. Militaru: *Extending structures for Lie algebras*, Monatshefte für Mathematik, **174**(2014), 169-193.
[3] A. Agore and G. Militaru: *Unified products for Leibniz algebras. Applications*, Linear Algebra Appl., **439**(2013), 2609-2633.
[4] F. Ammar and A. Makhlouf: *Hom Lie superlgebras and Hom-Lie admissible superalgebras*, J. Algebra, **324**(2010), 1513-1528.
[5] S. Bouarroudj and A. Makhlouf: *Hom-Lie Superalgebras in characteristic 2*, Mathematics,**11**(2023), No. 24, Art. No. 4955, 29 pages.
[6] J.T. Hartwig, D. Larsson and S.D. Silvestrov: *Deformations of Lie algebras using σ-derivation*, J. Algebra, **295**(2006), 314-361.
[7] V.G. Kac: *Lie superalgebras*, Adv. Math., **26**(1977), 8-96.
[8] D. Larsson and S.D. Silvestrov: *Quasi Hom-Lie algebras, Central extensions and 2-cocycle like identities*, J. Algebra, **288**(2005), 321-344.
[9] D. Larsson and S.D. Silvestrov: *Quasi Lie algebras*, in: Noncommutative Geometry and Representation Theory in Mathematical Physics, Contemp. Math., Amer. Math. Soc., Providence, **391**(2005), 241-248.
[10] D. Larsson and S.D. Silvestrov: *Quasi deformations of $sl_2(\mathbb{F})$ using twisted derivations*, Comm. Algebra, **35**(2007), No. 12, 4303-4318.
[11] D. Larsson and S.D. Silvestrov: *Graded quasi-Lie algebras*, Czechoslovak Journal of Physics, **55**(2005), 1473-1478.
[12] M. Pouye: *Hom Leibniz superalgebras with supersymmetric invariant nondegenerate bilinear forms*, J. Math. Res., **13**(2021), 73-89.

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