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A CONSTRUCTION OF FREE CO-ORDERED SEMIGROUP WITH APARTNESS

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Abstract. Semigroups with apartness have been the subject of study for over twenty years. In addition, semigroups with apartness ordered under a co-quasiorder (a co-order, res.) were also studied. A construction of a free semigroup with apartness was shown in 2011 by the author. In this paper, the author gives a construction of free semigroup with apartness ordered under a co-order and shows an important feature of such a constructed free co-ordered semigroup with apartness.

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