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APPLICATION OF AN IRRATIONAL TRIAL EQUATION METHOD TO HIGH-DIMENSIONAL NONLINEAR EVOLUTION EQUATIONS

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Abstract. An analytic technique, an irrational trial equation method, is applied to solve the (3+1)-dimensional potential-YTSF equation and the (2+1)-dimensional Broer-Kaup-Kupershmidt equations. Using this method, some exact travelling wave solutions to two high dimensional nonlinear evolution equations are obtained. This method provides us with a new way to obtain series solutions of such problems.

1. INTRODUCTION

As seeking exact solutions of nonlinear physics equations is important and interesting, many powerful methods have been presented. Among these are F-expansion method [1], homogeneous balance method [2], (G'/G)-expansion method [3] - [5], exp-function method [6] - [10], sine-cosine method [11,12], first integral method [13,14], Jacobi elliptic function method [15] and so on.

Recently, Liu proposed some trial equation methods which are different from those direct methods [16] - [19]. Then, Du defined a new method named irrational trial equation method and obtained new exact solutions to several nonlinear evolution equations in [20]. In this paper, we use the irrational trial equation method and give some exact solutions to the $(3+1)$-dimensional potential-YTSF equation [21] - [23]

$$-4u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_{xx}u_z + 3u_{yy} = 0, \tag{1.1}$$

and the $(2+1)$-dimensional Broer-Kaup-Kupershmidt (BKK) equations [24] - [31]

$$u_{ty} - u_{xxy} + 2(uu_x)_y + 2v_{xx} = 0, \tag{1.2}$$

$$v_t + v_{xx} + 2(uv)_x = 0. \tag{1.3}$$

2. IRRATIONAL TRIAL EQUATION METHOD

Do summarized the main steps for using the irrational trial equation method, as follows: Suppose that a nonlinear evolution equation is given by

$$N(u, u_t, u_{x_i}, u_{x_i x_i}, u_{tt}, u_{x_i t}, \ldots) = 0, \tag{2.1}$$

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where N is a polynomial in $u(x_i,t)$ and its various partial derivatives. Seeking for travelling wave solution of (2.1), taking $u(x_i,t) = U(\eta)$ and $\eta = \sum_i k_i x_i + wt$ leads to an ordinary differential equation as

$$M(U, wU', k_iU', w^2U'', wk_iU'', k_i^2U'', \ldots) = 0. \tag{2.2}$$

Take an irrational trial equation as follows

$$U' = \sum_{j=0}^{r_1} a_j U^j + \left(\sum_{j=0}^{r_2} b_j U^j\right)\sqrt{\sum_{j=0}^{r_3} c_j U^j}, \tag{2.3}$$

where a_j $(j = 0,\ldots,r_1), b_j$ $(j = 0,\ldots,r_2)$ and c_j $(j = 0,\ldots,r_3)$ are constants to be determined later. Using (2.3), we derive the following equation:

$$\begin{aligned} U'' = & \left(\sum_{i=1}^{k_1} ia_iU^{i-1}\right)\left(\sum_{i=0}^{k_1} a_iU^i\right) + \left(\sum_{i=0}^{k_2} b_iU^i\right)\left(\sum_{i=1}^{k_2} ib_iU^{i-1}\right)\left(\sum_{i=0}^{k_3} c_iU^i\right) \\ & + \frac{1}{2}\left(\sum_{i=0}^{k_2} b_iU^i\right)^2\left(\sum_{i=1}^{k_3} ic_iU^{i-1}\right) + \frac{1}{2}\left(\sum_{i=0}^{k_1} a_iU^i\right)\left(\sum_{i=0}^{k_2} b_iU^i\right)\left(\sum_{i=1}^{k_3} ic_iU^{i-1}\right)\left(\sum_{i=0}^{k_3} a_iU^i\right)^{-\frac{1}{2}} \\ & + \left[\left(\sum_{i=1}^{k_2} ib_iU^{i-1}\right)\left(\sum_{i=0}^{k_1} a_iU^i\right) + \left(\sum_{i=1}^{k_1} ia_iU^{i-1}\right)\left(\sum_{i=0}^{k_2} b_iU^i\right)\right]\sqrt{\sum_{i=0}^{k_3} c_iU^i} \end{aligned}$$

and other derivation terms such as U''', and so on. We determine the positive integers r_1, r_2 and r_3 in (2.3) by balancing the highest order derivatives and nonlinear terms in (2.3). Substituting (2.3) into (2.2), we have $P(U) + R(U)\sqrt{\sum_{j=0}^{r_3} c_jU^j} = 0$, where $P(U)$ and $R(U)$ are two polynomials of U.

Equating all coefficients of $P(U)$ and $R(U)$ to zero, we get a system of algebraic equations and solving these algebraic equations by Mathematica, we get the values of $a_0,\ldots,a_{r_1}, b_0,\ldots,b_{r_2}$ and $c_0,\ldots,c_{r_3}$. Finally, integrating (2.3) we can obtain the exact travelling wave solutions of (2.1).

3. APPLICATION TO THE $(3+1)$-DIMENSIONAL POTENTIAL-YTSF EQUATION

As described above the ansatz $u(x,t) = U(\eta)$, $\eta = kx + py + qz - wt$ is introduced in (1.1) to give the following nonlinear ordinary differential equation of fourth order:

$$k^3qU'''' + (4kw + 3p^2)U'' + 6k^2qU'U'' = 0. \tag{3.1}$$

Integrating (3.1) once and letting the integral constant be zero, we have

$$k^3qU''' + (4kw + 3p^2)U' + 3k^2q(U')^2 = 0. \tag{3.2}$$

Setting $U' = h(\eta)$ as a new dependent variable, a second order nonlinear ordinary differential equation results

$$k^3qh'' + 3k^2qh^2 + (4kw + 3p^2)h = 0. \tag{3.3}$$

Substituting (2.3) into (3.3) and balancing h'' and h^2 in (3.3), we find $2r_2 + r_3 = 3$ and $2r_1 < 3$. Then, we obtain $r_1 = r_2 = r_3 = 1$. So the irrational trial equation reduce to the following form

$$h' = a_0 + a_1h + (b_0 + b_1h)\sqrt{c_0 + c_1h}. \tag{3.4}$$

where a_j, b_j, c_j are the unknown parameters, for $j = 0, 1$. From (3.4), we obtain

$$h'' = \left(a_0 + a_1h + (b_0 + b_1h)\sqrt{c_0 + c_1h}\right) \times \left(a_1 + b_1\sqrt{c_0 + c_1h} + \frac{b_0c_1 + b_1c_1h}{2\sqrt{c_0 + c_1h}}\right). \tag{3.5}$$

Substituting h' and h'' into (3.3) yields the following algebraic equation $P(h)+R(h)\sqrt{c_0+c_1h}=0$, where $P(h) = k^3q\ [3a_1b_1c_1h^2 + (2a_1b_1c_0 + 2a_1b_0c_1 + 2a_0b_1c_1)h + a_1b_0c_0 + a_0b_1c_0 + a_0b_0c_1]$, and $R(h) = k^2q(3 + 2b_1^2c_1k)h^2 + (3p^2 + k^3q(a_1^2 + b_1^2c_0 + 3b_0b_1c_1) + 4kw)h + k^3q(a_0a_1 + b_0b_1c_0 + b_0^2c_1)$. Collecting the coefficients of h^i $(i = 0, 1, 2)$ in the case of $P(h) = 0$ and $R(h) = 0$ and setting each coefficient to zero, we obtain the following system of algebraic equations:

$$3a_1b_1c_1k^3q = 0, \tag{3.6}$$

$$k^3q(2a_1b_1c_0 + 2a_1b_0c_1 + 2a_0b_1c_1) = 0, \tag{3.7}$$

$$k^3q(a_1b_0c_0 + a_0b_1c_0 + a_0b_0c_1) = 0, \tag{3.8}$$

$$3k^2q + 2b_1^2c_1k^3q = 0, \tag{3.9}$$

$$3p^2 + a_1^2k^3q + b_1^2c_0k^3q + 3b_0b_1c_1k^3q + 4kw = 0, \tag{3.10}$$

$$k^3q(a_0a_1 + b_0b_1c_0 + b_0^2c_1) = 0. \tag{3.11}$$

Solving the algebraic (3.6) - (3.11) by Mathematica, we have the following cases of solutions:

Case 1. $a_0 = a_1 = b_0 = 0,\ b_1 = b_1,\ c_0 = -\dfrac{3p^2 + 4kw}{b_1^2k^3q},\ c_1 = -\dfrac{3}{2b_1^2k}.$

Integrating Eq. (3.4), we have

$$h(\eta) = -\frac{2(3p^2 + 4kw)}{3k^2q} \sec\left[\frac{1}{2k}\sqrt{\frac{3p^2 + 4kw}{kq}}\left(\eta - kC_1\sqrt{-2kq}\right)\right]. \tag{3.12}$$

Substituting (3.12) into $U' = h(\eta)$ and integrating this equation, we get a periodic solution of the $(3 + 1)$-dimensional potential-YTSF equation (1.1) as follows:

$$u(x, y, z, t) = C_2 - \frac{4}{3}\sqrt{\frac{3p^2 + 4kw}{kq}} \tan\left[\frac{1}{2k}\sqrt{\frac{3p^2 + 4kw}{kq}}\left(\eta - kC_1\sqrt{-2kq}\right)\right],$$

where $u(x, y, z, t) = U(\eta)$ and $\eta = kx + py + qz - wt$.

Case 2. $a_0 = a_1 = 0,\ b_0 = \dfrac{b_1(3p^2 + 4kw)}{3k^2q},\ c_0 = \dfrac{3p^2 + 4kw}{2b_1^2k^3q},\ c_1 = -\dfrac{3}{2b_1^2k}$, where b_1 is a free parameter. Integrating (3.5), we obtain

$$h(\eta) = \frac{3p^2 + 4kw}{3k^2q}\left\{1 + 2\tan\left[\frac{1}{2k}\sqrt{\frac{-3p^2 - 4kw}{kq}}\left(\eta + 3k^3qC_3\sqrt{-2kq}\right)\right]\right\}. \tag{3.13}$$

Substituting (3.13) into $U' = h(\eta)$ and integrating this equation, we find a periodic solution of Eq. (1.1) as follows:

$$U(\eta) = -\frac{M\eta}{3k^2q} + C_4 - \frac{4N}{3}\sqrt{-\frac{M}{kq}} \sec\left(\frac{1}{2k}\sqrt{-\frac{M}{kq}}\ \eta + 3k^2qC_3\sqrt{\frac{M}{2}}\right)\sin\left(\frac{1}{2k}\sqrt{-\frac{M}{kq}}\ \eta\right), \tag{3.14}$$

where $3p^2 + 4kw = M$ and $\sec\left(3k^2qC_3\sqrt{\dfrac{M}{2}}\right) = N$.

For simplicity if we take $C_3 = 0$, the solution (3.14) becomes

$$u(x, y, z, t) = C_4 - \frac{M\eta}{3k^2q} + \frac{4\sqrt{-Mkq}}{3kq}\tan\left(\frac{1}{2k}\sqrt{\frac{-M}{kq}}\eta\right),$$

where $u(x, y, z, t) = U(\eta)$ and $\eta = kx + py + qz - wt$.

4. APPLICATION TO THE $(2+1)$-DIMENSIONAL BROER-KAUP KUPERSHMIDT EQUATION

Using a complex variation η defined as $\eta = kx + qy + wt$, $u = \varphi(\eta)$ and $v = \phi(\eta)$, we can convert (1.2) and (1.3) into ordinary different equations, which read

$$qw\varphi'' - k^2 q\varphi''' + 2kq(\varphi\varphi')' + 2k^2\phi'' = 0, \tag{4.1}$$

$$w\phi' + k^2\phi'' + 2k(\varphi\phi)' = 0, \tag{4.2}$$

where the prime denotes the derivative with respect to η.

Integrating (4.1) twice and letting integral constants be zero, we obtain

$$\phi = \frac{q}{2}\varphi' - \frac{q}{2k}\varphi^2 - \frac{qw}{2k^2}\varphi. \tag{4.3}$$

Integrating (4.2) once, we have

$$w\phi + k^2\phi' + 2k\varphi\phi + c = 0. \tag{4.4}$$

where c is an integration constant.

Substituting (4.3) into (4.4), we get

$$qk^4\varphi'' - 2qk^2\varphi^3 - 2qwk\varphi^2 - qw^2\varphi + C_1 = 0. \tag{4.5}$$

where C_1 is a constant. Substituting (2.3) into (4.5) and balancing φ'' and φ^3, we have $2r_1 - 1 = 3$ and $2r_2 + r_3 - 1 \leq 3$. From here we obtain $r_1 = r_3 = 2$, $r_2 = 1$ and $r_1 = 2$, $r_2 = 0$, $r_3 = 4$. So the irrational trial equation reduce to the following form

If we take $r_1 = r_3 = 2, r_2 = 1$, then (2.3) becomes

$$\varphi' = a_0 + a_1\varphi + a_2\varphi^2 + (b_0 + b_1\varphi)\sqrt{c_0 + c_1\varphi + c_2\varphi^2} \tag{4.6}$$

where a_j, b_j, c_j are the unknown parameters. From (4.6), we get

$$\begin{aligned}\varphi'' = &\left(a_0 + a_1\varphi + a_2\varphi^2 + (b_0 + b_1\varphi)\sqrt{c_0 + c_1\varphi + c_2\varphi^2}\right)\\ &\times\left(a_1 + 2a_2\varphi + b_1\sqrt{c_0 + c_1\varphi + c_2\varphi^2} + \frac{(b_0 + b_1\varphi)(c_1 + 2c_2\varphi)}{2\sqrt{c_0 + c_1\varphi + c_2\varphi^2}}\right).\end{aligned}$$

Substituting φ' and φ'' into (4.5), we obtain the following equation

$$P(\varphi) + R(\varphi)\sqrt{c_0 + c_1\varphi + c_2\varphi^2} = 0.$$

Collecting the coefficients of φ^j $(i = 0, 1, \ldots, 5)$ in the case of $P(\varphi) = 0$ and $R(\varphi) = 0$ and equating each coefficient to zero, we obtain the following system of algebraic equations:

$$8a_2b_1c_2k^4q = 0, \tag{4.7}$$

$$7a_2b_1c_1k^4q + 6a_2b_0c_2k^4q + 6a_1b_1c_2k^4q = 0, \tag{4.8}$$

$$6a_2b_1c_0k^4q + 5a_2b_0c_1k^4q + 5a_1b_1c_1k^4q + 4a_1b_0c_2k^4q + 4a_0b_1c_2k^4q = 0, \tag{4.9}$$

$$4a_2b_0c_0k^4q + 4a_1b_1c_0k^4q + 3a_1b_0c_1k^4q + 3a_0b_1c_1k^4q + 2a_0b_0c_2k^4q = 0, \tag{4.10}$$

$$2a_1b_0c_0k^4q + 2a_0b_1c_0k^4q + a_0b_0c_1k^4q = 0, \tag{4.11}$$

$$-4k^2q + 4a_2^2k^4q + 4b_1^2c_2k^4q = 0, \tag{4.12}$$

$$6a_1a_2k^4q + 3b_1^2c_1k^4q + 6b_0b_1c_2k^4q - 4kqw = 0, \tag{4.13}$$

$$2a_1^2k^4q + 4a_0a_2k^4q + 2b_1^2c_0k^4q + 4b_0b_1c_1k^4q + 2b_0^2c_2k^4q - 2qw^2 = 0, \tag{4.14}$$

$$2c + 2a_0a_1k^4q + 2b_0b_1c_0k^4q + b_0^2c_1k^4q = 0. \tag{4.15}$$

Solving the above algebraic equations (4.7) - (4.15), we obtain:

$$a_0 = a_1 = a_2 = 0,\ b_0 = b_0,\ b_1 = b_1,\ c_0 = c_0,\ c_1 = c_1,\ c_2 = c_2.$$

Integrating (4.6) and letting integral constant be zero, we have

$$\varphi(\eta) = \frac{-b_0A^4 + 4A^5e^{A\eta} + 2b_1A(4c_1A + b_0b_1B)e^{2A\eta} + 4b_1^2A^3Be^{3A\eta} - b_0b_1^4B^2e^{4A\eta}}{b_1A^2 - 2A(8c_2A + b_1^2B)e^{2A\eta} + b_1^4B^2e^{4A\eta}}$$

where $u(x, y, t) = \varphi(\eta)$ and $\eta = kx + qy + wt$.

If we take $k_1 = 2, k_2 = 0, k_3 = 4$, then (2.3) becomes

$$\varphi' = a_0 + a_1\varphi + a_2\varphi^2 + b_0\sqrt{c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4} \tag{4.16}$$

where a_j, b_j, c_j are the parameters to be determined. Furthermore, from (4.16), we have

$$\begin{aligned}\varphi'' &= \left(a_0 + a_1\varphi + a_2\varphi^2 + b_0\sqrt{c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4}\right)\\ &\quad\times\left(a_1 + 2a_2\varphi + \frac{b_0(c_1 + 2c_2\varphi + 3c_3\varphi^2 + 4c_4\varphi^3)}{2\sqrt{c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4}}\right).\end{aligned}$$

Using the solution procedure of irrational trial equation method, we obtain following results:

$$a_0 = a_1 = a_2 = 0,\ c_1 = -\frac{2C_1}{b_0^2k^4q},\ c_2 = \frac{w^2}{b_0^2k^4},\ c_3 = \frac{4w}{3b_0^2k^3},\ c_4 = \frac{1}{b_0^2k^2},$$

where b_0 and c_0 are free parameters. In this case, (4.16) becomes

$$\varphi' = \sqrt{b_0^2c_0 - \frac{2C_1}{k^4q}\varphi + \frac{w^2}{k^4}\varphi^2 + \frac{4w}{3k^3}\varphi^3 + \frac{1}{k^2}\varphi^4}. \tag{4.17}$$

According to the known exact solutions of the generalized Riccati equation [32,33], the solitary wave solution of (4.17) can be written as

$$\varphi(\eta) = \frac{-\dfrac{32C_2w^3}{3k^7}\mathrm{sech}^2\left(\dfrac{w}{k^2}\eta\right)}{\dfrac{64w^2}{9k^6}\mathrm{sech}^2\left(\dfrac{2w}{k^2}\eta\right) + \left(\dfrac{64w^2}{9k^6} + B\right)\tanh\left(\dfrac{w}{k^2}\eta\right) + \dfrac{64w^2}{9k^6} - B},$$

where C_2 is an integration constant and $B = C_2^2\left(\dfrac{4w^2}{k^6} - \dfrac{16w^2}{9k^6}\right)$.

Thus the BKK equation has the following solitary wave solution

$$u(x, y, t) = \frac{-\dfrac{32C_2w^3}{3k^7}\mathrm{sech}^2\left(\dfrac{w}{k^2}\eta\right)}{\dfrac{64w^2}{9k^6}\mathrm{sech}^2\left(\dfrac{2w}{k^2}\eta\right) + \left(\dfrac{64w^2}{9k^6} + B\right)\tanh\left(\dfrac{w}{k^2}\eta\right) + \dfrac{64w^2}{9k^6} - B},$$

where $u(\eta) = \varphi$ and $\eta = kx + qy + wt$.

5. CONCLUSION

In this paper, we have obtained exact solutions for the $(3 + 1)$-dimensional potential YTSF equation and the $(2 + 1)$-dimensional BKK equations by employing the irrational trial equation method. We believe that new exact solutions to nonlinear evolution equations will be found by this method. Also this method can be extended to other high-dimensional nonlinear evolution equations.

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