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SOLUTIONS OF SECOND ORDER NONHOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS WITH MEROMORPHIC COEFFICIENTS

MAAMAR ANDASMAS AND BENHARRAT BELAÏDI

Abstract. In this paper, we mainly consider the growth and the oscillation of solutions of the second order nonhomogeneous linear differential equations

$$f'' + h_1(z)\, e^{P(z)} f' + h_0(z)\, e^{Q(z)} f = F(z),$$

where $P(z)$, $Q(z)$ are nonconstant polynomials such that $\deg(P) \neq \deg(Q)$ and $h_j(z) \not\equiv 0$ $(j = 0, 1)$, $F(z)$ are meromorphic functions of finite order having only finitely many poles such that $\rho(h_0) < \deg(Q)$, $\rho(h_1) < \deg(P)$ and $\rho(F) < \max\{\deg(Q), \deg(P)\}$. We show that all transcendental meromorphic solutions f have an infinite order and we give an estimate of their hyper-order. In the last, we give an estimation for the exponent of convergence of fixed points of solutions and their 1st, 2nd derivatives.

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University of Mostaganem (UMAB)
Department of Mathematics
Laboratory of Pure and Applied Mathematics
B. P. 227 Mostaganem (Algeria)
E-mail address: maamaths73@yahoo.fr

University of Mostaganem (UMAB)
Department of Mathematics
Laboratory of Pure and Applied Mathematics
B. P. 227 Mostaganem (Algeria)
E-mail address: belaidi@univ-mosta.dz (corresponding author)