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ON THE EXISTENCE OF SOLUTIONS FOR NONLINEAR FRACTIONAL q-DIFFERENCE EQUATIONS WITH q-INTEGRAL BOUNDARY CONDITIONS

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Abstract. In this paper, we investigate the existence of solutions for nonlinear fractional q-difference equation ${}^cD_q^\alpha u(t) = f(t, u(t), D_q u(t))$ with the boundary value conditions $\lambda_1 u(0) + \mu_1 D_q u(0) = \eta_1 I_q^\beta u(\xi_1)$ and $\lambda_2 u(1) + \mu_2 D_q u(1) = \eta_2 I_q^\beta u(\xi_2)$, where $1 \leq \alpha \leq 2$, $0 < q < 1$ and ${}^cD_q^\alpha$ denotes the fractional q-derivative of the Caputo type of order α. By applying the some fixed point theorems, we prove our main results. Also, an example is presented.

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