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STRONG CONVERGENCE THEOREM FOR NOOR ITERATIONS OF A FINITE FAMILY OF ASYMPTOTICALLY QUASI-NONEXPANSIVE TYPE MAPPINGS IN CAT(0) SPACES

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Abstract. In this paper, we study strong convergence for Noor iterations of a finite family of asymptotically quasi-nonexpansive type mappings in the framework of CAT(0) spaces. The said iteration scheme includes modified Mann and Ishikawa iterations, the three-step iteration scheme of Xu and Noor and the scheme of Khan, Domlo and Fukhar-ud-din as special cases. Also, we establish some strong convergence theorems using semi-compactness and condition (A). Our results extend and generalize the previous works from the current existing literature.

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