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MINTY-TYPE LEMMAS AND GENERALIZED VECTOR EQUILIBRIUM-LIKE PROBLEMS IN NORMED SPACES

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Abstract. In this paper, we study two vector versions of Minty-type lemmas for generalized vector equilibrium-like problems (for short, GVELPs) in normed spaces, and obtain the existence theorems for three kinds of GVELPs. The results presented in this paper are the extension and improvement of the corresponding results of other authors.

REFERENCES

[1] Q.H. Ansari, A.H. Siddiqi and J.C. Yao: Generalized vector variational-like inequalities and their scalarizations, in *Vector Variational Inequalities and Vector Equilibria* (F. Giannessi (Ed.)), Kluwer Academic Publishers, Dordrecht, Holland, 2000, pp. 17-37.

[2] C. Baiocchi and A. Capelo: *Variational and Quasi-Variational Inequalities*, Applications to Free Boundary Problems, Weily, New York, 1984.

[3] G.Y. Chen: *Existence of solutions for a vector variational inequality: An extension of Hartman-Stampacchia theorem*, J. Optim. Theory Appl., **74**(1992), 445-456.

[4] G.Y. Chen and X.Q. Yang: *The vector complementarity problem and its equivalence with the weak minimal element*, J. Math. Anal. Appl., **153**(1990) 136-158.

[5] Y. Chiang: *Semicontinuous mapping in t.v.s. with applications to mixed vector variational-like inequalities*, J. Global Optim., **32**(2005), 467-486.

[6] A. Daniilidis and N. Hadjisavvas: Existence theorems for vector variational inequalities, Bull. Aust. Math. Soc., **54**(1996), 473-481.

[7] M. Fakhar and J. Zafarani: *Generalized vector equilibrium problems for pseudomonotone bifunctions*, J. Optim. Theory Appl., **126**(2005), 109-124.

[8] Y.P. Fang and N.J. Huang: *Variational-like inequalities with generalized monotone mapping in Banach spaces*, J. Optim. Theory Appl., **118**(2003), 327-338.

[9] F. Giannessi: Theorems of the alternative quadratic programs and complementarity problems, in *Variational Inequalities and Complementarity Problems* (R.W. Cottle, F. Giannessi, J.L. Lions (Eds)), John Wiley, Chichester, 1980, pp. 151-186.

[10] F. Giannessi: *On Minty Variational Principle, New Trends in Mathematical Programming*, Kluwer Academic Publishers, Dordrecht, Netherlands, 1997.

[11] F. Giannessi (Ed.): *Vector Variational Inequalities and Vector Equilibria*, Kluwer Academic Publishers, Dordrecht, Holland, 2000.

[12] F. Giannessi and A. Maugeri: *Variational Inequalities and Network Equilibrium Problems*, Plenum Press, New York, 1995.

[13] F. Giannessi and A. Maugeri: *Variational Analysis and Applications, Non-convex Optimization and Its Applications*, Vol. 79, Springer, New York, 2005.

[14] N.J. Huang and Y.P. Fang: *On vector variational inequalities in reflexive Banach spaces*, J. Global Optim., **32**(2005), 495-505.

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[15] L.C. Ceng and S. Huang: *Existence theorems for generalized vector variational inequalities with a variable ordering relation*, J. Global Optim., **46**(2010), 521-535.
[16] T. Jabarootian and J. Zafarani: *Generalized vector variational-like inequalities*, J. Optim. Theory Appl., **136**(2008), No. 1, 15-30.
[17] M.F. Khan and Salahuddin: *On generalized vector variational-like inequalities*, Nonlinear Anal., **59**(2004), 879-889.
[18] I.V. Konnov and J.C. Yao: *On the generalized vector variational inequality problems*, J. Math. Anal. Appl., **206**(1997), 42-58.
[19] B.S. Lee and G.M. Lee: *A vector version of Minty lemma and applications*, Appl. Math. Lett., **12**(1999), 43-50.
[20] L.J. Lin: *Pre-vector variational inequalities*, Bull. Aust. Math. Soc., **53**(1996), 63-70.
[21] S.B. Jr. Nadler: *Multi-valued contraction mappings*, Pacific J. Math., **30**(1969), No. 2, 475-488.
[22] X.Q. Yang: *Vector variational inequality and vector pseudolinear optimization*, J. Optim. Theory Appl., **95**(1997), 729-734.
[23] X.Q. Yang and C.J. Goh: *On vector variational inequality application to vector traffic equilibria*, J. Optim. Theory Appl., **95**(1997), 431-443.
[24] L.C. Ceng and J.C. Yao: *Existence of solutions of generalized vector variational inequalities in reflexive Banach spaces*, J. Global Optim., **36**(2006), 483-497.
[25] M. Chinaie, T. Jabarootian, M. Rezaie and J. Zafarani: *Minty's lemma and vector variational-like inequalities*, J. Global Optim. **40**(2008), 463-473.
[26] L.C. Ceng, G. Mastroeni and J.C. Yao: *Existence of solutions and variational principles for generalized vector systems*, J. Optim. Theory Appl., **137**(2008), No. 3, 485-495.
[27] L.C. Zeng and J.C. Yao: *Generalized Minty's lemma for generalized vector equilibrium problems*, Appl. Math. Lett., **20**(2007), No. 1, 32-37.
[28] L.C. Zeng and J.C. Yao: *An existence result for generalized vector equilibrium problems without pseudomonotonicity*, Appl. Math. Lett., **19**(2006), No. 12, 1320-1326.
[29] L.C. Ceng, G.Y. Chen, X.X. Huang and J.C. Yao: *Existence theorems for generalized vector variational inequalities with pseudomonotonicity and their applications*, Taiwanese J. Math., **12**(2008), No. 1, 151-172.
[30] L.C. Ceng and S. Kum: *On generalized vector implicit variational inequalities and complementarity problems*, Taiwanese J. Math., **11**(2007), No. 3, 621-636.
[31] L.C. Ceng, P. Cubiotti and J.C. Yao: *Existence of vector mixed variational inequalities in Banach spaces*, Nonlinear Anal., **70**(2009), No. 3, 1239-1256.
[32] L.C. Ceng, S. Schaible and J.C. Yao: *Existence of solutions for generalized vector variational-like inequalities*, J. Optim. Theory Appl., **137**(2008), No. 1, 121-133.
[33] L.C. Zeng and J.C. Yao: *Existence theorems for variational inequalities in Banach spaces*, J. Optim. Theory Appl., **132**(2007), No. 2, 321-337.

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