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EXISTENCE AND NON-EXISTENCE OF SOLUTION FOR A SINGULAR NONLINEAR DIRICHLET PROBLEM INVOLVING THE p(x)-LAPLACE OPERATOR

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Abstract. Using variational arguments we investigate the p(x)-Laplace equation with singular non-linearities in order to show some existence and non-existence results.

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