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THE MODIFIED SIMPLE EQUATION METHOD AND THE METHOD OF SOLITON ANSATZ FOR SOLVING THE GENERALIZED NONLINEAR SCHRÖDINGER EQUATION WITH A SOURCE

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Abstract. The modified simple equation method and the method of soliton ansatz for solving nonlinear partial differential equations (PDEs) are presented. Based on these two methods, we obtain the exact solutions and the bright-dark soliton solutions with parameters of the generalized nonlinear Schrödinger equation with a source. When these parameters are taken special values, the solitary wave solutions are derived from the exact solutions. Comparison between our new results and the well-known results is obtained.

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